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ON THE TRISECTION OF AN ANGLE.

By E. E. WHITE, M. E., Harvard University.

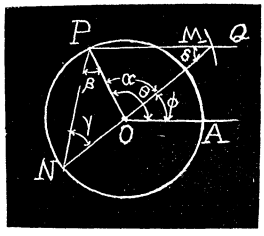
The two following approximate constructions for the trisection of an angle may be interesting. They are, of course, essentially methods of trial and error, since it is only after placing the straight edge in position, passing through the given point, that one can tell whether the position is that desired. The straight-edge must be continually shifted till all the conditions are fulfilled, as nearly as can be told by eye, and mathematically speaking the methods are therefore only approximate.

While it is well known that an angle cannot be trisected by *strict* geometrical construction, nevertheless there are many interesting and practically useful approximate constructions, of which the following are examples. The principle of neither construction is new, though the author devised the first method independently. Four years ago Mr. John J. Quinn made two linkages which involve these constructions, and the linkages in turn depend upon the limaçon. Further, the construction using a graduated scale is an application of the method devised by Archimedes, known as the "Method of Insertions."

CONSTRUCTION I.

Given angle θ, to trisect the angle.

With the vertex, O, as a center, describe a circle of any radius, intersecting the sides of the angle at P and A. Through P draw the line PQ parallel to OA by the usual method. Pass a straight-edge through O, and adjust the position of the straight-edge so that its intersection N with the circle O, and its intersection M with the line PQ shall be equidistant from P (as tested by compasses). Draw the line NOM, which will trisect the angle.



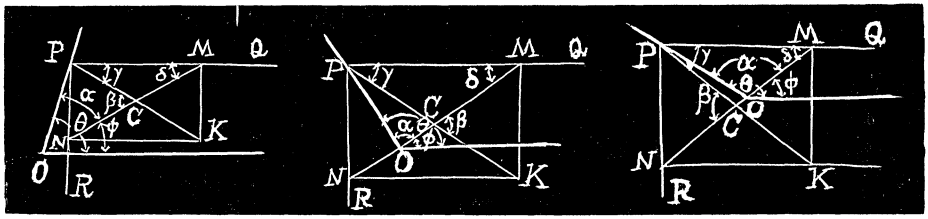
Proof. Draw the line PN.

Then $\theta=\alpha+\phi=\beta+\gamma+\phi=2\gamma+\phi=2\delta+\phi=2\phi+\phi=3\phi$.

CONSTRUCTION II.

Given angle θ, to trisect the angle.

Through any point P on one side of the angle draw PQ parallel and



PR perpendicular to the opposite side of the angle. Mark off a distance equal to $2\overline{OP}$ on a straight-edge passing through O, and adjust the straight-edge so that one mark falls on line PQ at M, and the other on line PR at N. Draw the line NOM, which will trisect the angle.

Proof: Complete the rectangle $PMKN$, and draw the other diagonal PK. Then $\overline{OP}=\frac{1}{2}\overline{MN}=\frac{1}{2}\overline{PK}=\overline{PC}=\overline{CM}$.

Hence, $\theta=\alpha+\phi=\beta+\phi=\gamma+\delta+\phi=2\delta+\phi=2\phi+\phi=3\phi$.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

283. **Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Solve $w+x+y+z=4a$, $w^2+x^2+y^2+z^2=4a^2+4b^2$, $w^3+x^3+y^3+z^3=4a^3+12ab^2$, $w^4+x^4+y^4+z^4=4a^4+4b^4+4c^4+24a^2b^2$.

Solution by DR. L. E. DICKSON, Associate Professor of Mathematics, The University of Chicago.

The following method applies equally well to the corresponding equations with arbitrary constant terms. We are given s_1, s_2, s_3, s_4, where s_n is the sum of the nth powers of w, x, y, z. Hence the latter are, by Newton's identities, the roots of the following quartic:

$$\xi^4-4a\xi^3+(6a^2-2b^2)\xi^2+(4ab^2-4a^3)\xi+a^4+b^4-2a^2b^2-c^4=0.$$

To obtain the reduced quartic, set $\xi=\eta+a$. Then

$$\eta^4-2b^2\eta^2+b^4-c^4=0, \quad (\eta^2-b^2)^2=c^4.$$

Hence, the 24 sets of solutions are given by the arrangements of $a\pm\sqrt{(b^2\pm c^2)}$.

Similarly solved by G. B. M. Zerr and J. Scheffer.

GEOMETRY.

312. **Proposed by F. H. SAFFORD, Ph. D., The University of Pennsylvania, Philadelphia, Pa.**

A variable circle passes through a fixed point and is tangent to a given circle. If a diameter of the first circle passes through the fixed point, find the locus of its other extremity.